

# A Comment on a Fivebrane Term in Superalgebra

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## Abstract

We review our recent discussion of fivebrane central terms that appear in the space-time superalgebra in  $D = 10$  provided that the space-time supercharges are taken in non-canonical pictures. We correct the mistake contained in the original version of the earlier paper 9703008 which suggested that the naive picture-changing of the superalgebra gives rise to the non-perturbative five-form term. This term vanishes because of gamma-matrix identity in  $D = 10$ , as has been pointed out by Berkovits in his recent paper.

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In this letter, we review the derivation of the five-form term, in the [1], showing that this term vanishes due to the gamma-matrix identity in ten dimensions. We will argue, however, that the fivebrane is present in the picture-changed superalgebra in an indirect way, as a total derivative three-form term. This tree-form corresponds to a threebrane which is not fundamental, but rather is an intersection of two fivebranes. We start with recalling the operator product expansions of the space-time spin operators with themselves and with the worldsheet NSR fermions [2]. We have:

$$\begin{aligned} \Sigma_\alpha(z)\Sigma^\beta(w) \sim & \frac{\delta_\alpha^\beta}{(z-w)^{5/4}} + \frac{(\Gamma^m)_\alpha^\beta}{(z-w)^{3/4}}\psi_m + \frac{(\Gamma^{m_1 m_2})_\alpha^\beta}{(z-w)^{1/4}}\psi_{m_1}\psi_{m_2} + \\ & + \sum_p \frac{(\Gamma^{m_1 \dots m_p})_\alpha^\beta}{(z-w)^{5/4-p/2}}\psi_{m_1} \dots \psi_{m_p} + \text{derivatives} \end{aligned} \quad (1)$$

Here the index  $m$  runs from 0 to 9, and  $\Gamma^m$  are 32x32 ten-dimensional gamma-matrices. The fermionic indices  $\alpha, \beta, \dots$  are alternated by the charge conjugation matrix  $C_{\alpha\beta} = (\Gamma^0)_{\alpha\beta}$ . The O.P.E. between  $\Sigma$  and  $\psi$  is given by: [3,2]:

$$\psi^m(z)\Sigma_\alpha(w) \sim \frac{(\Gamma^m)_\alpha^\beta \Sigma_\beta(w)}{(z-w)^{1/2}} \quad (2)$$

Alternatively, by raising the index  $\alpha$  simultaneously in the l.h.s and the r.h.s. of the last equation ( by multiplying both the l.h.s. and the r.h.s. by  $\Gamma^0$  from the left), we can write the O.P.E. as

$$\psi^m(z)\Sigma^\alpha(w) \sim \frac{(\Gamma^m)^{\alpha\beta} \Sigma_\beta(w)}{(z-w)^{1/2}} = \frac{(\Gamma^m)^\alpha_\beta \Sigma^\beta(w)}{(z-w)^{1/2}} \quad (3)$$

Next, the expression for the space-time supercharges in the canonical picture is given by:

$$Q_\alpha = \oint \frac{dz}{2i\pi} e^{-1/2\phi} \Sigma_\alpha(z) \quad (4)$$

where  $\phi$  is bosonized superconformal ghost field [3]. As is easy to check, the computation of the anticommutator of two supercharges in this canonical picture, by using the O.P.E. (1) gives the standard result:

$$\{Q_\alpha, Q^\beta\} = (\Gamma^m)_\alpha^\beta P_m \quad (5)$$

, where  $P_m = \oint \frac{dz}{2i\pi} e^{-\phi} \psi_m(z)$  is the momentum operator in the  $-1$ -picture. Now, we would like to consider the same anticommutator with the supercharges  $Q_\alpha, Q^\beta$  being taken in the

non-canonical  $+1/2$ -picture. These supercharges are obtained from those of the formula (5) by the picture-changing transformation, implemented by the picture-changing operator

$$: \Gamma_1 := e^\phi \psi^m \partial X_m + \text{ghosts} \quad (6)$$

The ghost terms will be dropped in our calculation since they produce the contributions not significant for the correlation functions. By using the O.P.E's (2), (3), we find that the expressions for the supercharges in the  $+1/2$ -picture are given by:

$$\begin{aligned} Q_\alpha^{(+1/2)} &= \oint \frac{dz}{2i\pi} e^{1/2\phi} (\Gamma^m)_\alpha{}^\beta \Sigma_\beta \partial X_m \\ Q^{\alpha(+1/2)} &= \oint \frac{dz}{2i\pi} e^{1/2\phi} (\Gamma^m)^\alpha{}_\beta \Sigma^\beta \partial X_m \end{aligned} \quad (7)$$

The evaluation of the anticommutator, again by using the O.P.E. (1) gives:

$$\begin{aligned} \{Q_\alpha^{(+1/2)}, Q^{\beta(+1/2)}\} &= (\Gamma^m)_\alpha{}^\beta P_m^{(+1)} + \\ &+ (\Gamma^n)_\alpha{}^\gamma (\Gamma^{m_1 \dots m_5})_\gamma{}^\delta (\Gamma_n)^\beta{}_\delta Z_{m_1 \dots m_5} \end{aligned} \quad (8)$$

Here  $P_m^{(+1)}$  is the momentum operator in the  $+1$ -picture (the expression for it is given in [1]) and the five-form central charge  $Z$  is found in [1] to be equal to:

$$Z_{m_1 \dots m_5} = e^\phi \psi_{m_1} \dots \psi_{m_5} \quad (9)$$

Let us analyze now the gamma-matrix factor in the five-form term in the last equation. We have:

$$\begin{aligned} (\Gamma^n)_\alpha{}^\gamma (\Gamma^{m_1 \dots m_5})_\gamma{}^\delta (\Gamma_n)^\beta{}_\delta &= (\Gamma^n)_{\alpha\gamma} (\Gamma^{m_1 \dots m_5})^{\gamma\delta} (\Gamma_n^T)_\delta{}^\beta = \\ &= (\Gamma^n)_{\alpha\gamma} (\Gamma^{m_1 \dots m_5})^{\gamma\delta} (\Gamma_n^T)_{\delta\rho} \Gamma_0^{\rho\beta} = \\ &= 2(\Gamma^{m_1 \dots m_5})_\alpha{}^\beta \end{aligned} \quad (10)$$

But for the gamma-matrices with both indices down  $\Gamma^m = (\Gamma^m)^T$ . Therefore the gamma-matrix factor in front of the five-form term is proportional to  $\Gamma^m \Gamma^{m_1 \dots m_5} \Gamma_m$ , and this factor vanishes in  $D = 10$ . This concludes our analysis of the gamma-matrix factor in the fivebrane term. The fivebrane term, as we see, does appear in the superalgebra with the supercharges taken in the non-canonical picture. We conclude by adding few observations about the space-time superalgebras in non-canonical pictures. While the five-form term vanishes in the anticommutator  $\{Q^{+\frac{1}{2}}, Q^{+\frac{1}{2}}\}$ , there is another p-form term, namely, the

three-form that appears in the space-time *supercurrent* algebra. Being the total derivative, it is proportional to:

$$Z_{m_1\dots m_3} \sim \partial(e^\phi \psi_{m_1}\dots\psi_{m_3}) \quad (11)$$

It is known that two fivebranes intersect over a threebrane . The total derivative seems to be related to the fact that the threebrane is not fundamental, but rather is an intersection of two fivebranes. Therefore, in an indirect way, the fivebrane does appear in the picture-changed superalgebra. Therefore the information about the dynamics of intersecting branes may be hidden in the concrete structure of space-time supercurrent algebra. I'm grateful to N.Berkovits and E.Witten for pointing out to me the error in the original version of my paper hep-th/9703008, in which the vanishing  $\Gamma$ -matrix factor was overlooked. The argument about the vanishing if the fivebrane term has been given earlier in [4]

## References

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